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METHODOLOGY OF TEACHING MATHEMATICS

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TEACHING FUNCTIONAL ANALYSIS THROUGH ENGINEERING PROBLEMS

Konul Hamid Aliyeva
associate professor at Sumgayit State University,
Doctor of Philosophy in Mathematics
ORCID: 0000-0003-0316-947X
E-mail: konul.aliyeva@sdu.edu.az
Leyla Vidadi Nasirova
teacher at Sumgayit State University,
Doctor of Philosophy in Mathematics
ORCID: 0000-0002-9480-1372
E-mail: leyla.nasirova@sdu.edu.az,
Ulkar Rafiq Babayeva
Sumgayit State University,
ORCID: 0000-0002-2484-8730
E-mail: ulkar.babayeva@sdu.edu.az
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Açar sözlər: *mühəndislik təhsili, operator nəzəriyyəsi, konstruktiv təlim, problem əsaslı öyrənmə, layihə əsaslı öyrənmə, STEM təhsili, rəqəmsal təlim texnologiyaları.*

Ключевые слова: *инженерное образование, теория операторов, конструктивистское обучение, проблемно-ориентированное обучение, проектно-ориентированное обучение, STEM-образование, цифровые образовательные технологии.*

The rapid advancement of science and technology in the twenty-first century has led to significant changes in the content and structure of engineering education. The development of digital technologies, the widespread adoption of artificial intelligence systems, the processing of big data, the implementation of automated control systems, and the emergence of high-tech industries require engineers to possess a deeper and more comprehensive mathematical background. Under these conditions, mathematical modeling has become one of the fundamental components of engineering practice and professional activity. For this reason, functional analysis occupies an important place in engineering education.

However, functional analysis is widely regarded as one of the most challenging mathematical disciplines for students to master. The main reasons for this include:

- the high level of abstraction of its concepts;
- the difficulty of intuitively understanding infinite-dimensional spaces;
- insufficient integration with previously acquired mathematical knowledge;
- the inadequate connection between theoretical material and practical applications;
- the predominance of traditional lecture-based teaching methods.

These challenges necessitate a reconsideration of the methodology used in teaching functional analysis and highlight the need for its enhancement through the application of modern pedagogical approaches.

Scientific novelty of the problem

The primary objective of this study is to identify the scientific and methodological foundations for the effective teaching of functional analysis in engineering programs and to develop an instructional model based on modern educational technologies.

To achieve this objective, the following tasks were established:

1. To determine the role and significance of functional analysis in engineering education;
2. To investigate the potential of contemporary pedagogical approaches;
3. To identify ways of integrating functional analysis topics with engineering problems;
4. To evaluate the role of digital technologies in the teaching and learning process;
5. To analyze assessment and monitoring mechanisms;
6. To develop recommendations for the practical implementation of the proposed instructional model.

Research Methods

The following research methods were employed in the preparation of this article:

- systematic analysis of scientific and pedagogical literature;
- comparative analysis;
- content analysis;
- modeling methodology;
- observation and generalization;
- examination of international educational practices.

Problem statement

Form of the equation Functional analysis serves as a bridge between classical mathematical analysis and modern applied mathematics. This field encompasses the theory of Banach and Hilbert spaces, linear operators, spectral theory, and variational methods.

The range of applications of functional analysis in engineering is remarkably broad and includes numerous areas of modern science and technology:

Solution of the problem Example

1.Linear Spaces

The concept of a linear space constitutes the starting point of functional analysis.

Definition:

X be a set on which the following operations are defined:

$$\boldsymbol{x+y \in X}$$

Scalar multiplication $\boldsymbol{\alpha x \in X}$ If the axioms of a linear space are satisfied under these operations, then (X) is called a **linear space** (or **vector space**)

Example 1: $\boldsymbol{R^n}$ The euclidean space.

Example 2: $\boldsymbol{C[a,b]}$ The space of all continuous functions defined on the interval $[\boldsymbol{a},\boldsymbol{b}]$

Example 3: $\boldsymbol{P_n}$The space of all polynomials of degree not exceeding (n).

Example 4: $\boldsymbol{L^2(a,b)}$The space of square-integrable functions.

Among these spaces, $\boldsymbol{C[a,b]}$ and $\boldsymbol{L^2(a,b)}$are particularly important in engineering applications.

For example, the mathematical models used in:

- signal processing;
- heat conduction;
- electromagnetic phenomena;
- communication systems;

are frequently formulated within these functional spaces.

2. Normed Spaces

To measure the distance between elements of a linear space, the concept of a norm is introduced.

Definition 2. $\boldsymbol{X}$ be a linear space. A function $\| \boldsymbol{x} \|: \boldsymbol{X} \to \boldsymbol{R}$ is called a norm if it satisfies the following conditions:

Positivity

$$\| \boldsymbol{x} \| \geq \mathbf{0}$$

and

$$\| \boldsymbol{x} \| = \mathbf{0} \iff \boldsymbol{x} = \mathbf{0}$$

Homogeneity

$$\| \boldsymbol{\alpha x} \| = | \boldsymbol{\alpha} | \| \boldsymbol{x} \|$$

Triangle Inequality

$\| \boldsymbol{x} + \boldsymbol{y} \| \leq \| \boldsymbol{x} \| + \| \boldsymbol{y} \|$A linear space equipped with a norm is called a normed space.

3.Fundamental Norms. Euclidean Norm $\| \boldsymbol{x} \|_2 = \left(\sum_{i=1}^{n} \boldsymbol{x}_i^2 \right)^{1/2}$

Maximum Norm $\| \boldsymbol{x} \|_\infty = \mathbf{max} \; | \boldsymbol{x}_i |$These norms are widely used in optimization, data analysis, machine learning, signal processing, and numerous engineering applications.

4. Hilbert Spaces. A Hilbert space is one of the most fundamental concepts in functional analysis.

Definition. Let a linear space be equipped with an inner product

$$\langle \boldsymbol{x}, \boldsymbol{y} \rangle$$

If the space is complete with respect to the norm induced by this inner product, then it is called a **Hilbert space**.

Properties of the Inner Product

Symmetry

$\langle \boldsymbol{x}, \boldsymbol{y} \rangle = \langle \boldsymbol{y}, \boldsymbol{x} \rangle$ (for real spaces; in complex spaces, the conjugate symmetry property holds).

Linearity

$$\langle \boldsymbol{ax} + \boldsymbol{by}, \boldsymbol{z} \rangle = \boldsymbol{a} \langle \boldsymbol{x}, \boldsymbol{z} \rangle + \boldsymbol{b} \langle \boldsymbol{y}, \boldsymbol{z} \rangle$$

Positivity

$$\langle \boldsymbol{x}, \boldsymbol{x} \rangle \geq \mathbf{0}$$

with equality if and only if (x=0).

Norm induced by the inner product

The norm in a Hilbert space is defined by

$$\| \boldsymbol{x} \| = \sqrt{\langle \boldsymbol{x}, \boldsymbol{x} \rangle}$$

Classical examples of Hilbert Spaces

Euclidean Space

$$\boldsymbol{R^n}$$

equipped with the standard inner product.

Space of Square-Integrable functions

$$\boldsymbol{L^2(a, b)}$$

where the inner product is defined as

$$\langle \boldsymbol{f}, \boldsymbol{g} \rangle = \int_a^b \boldsymbol{f(x)g(x)\,dx}$$

Engineering applications of hilbert spaces

Hilbert spaces play a crucial role in numerous areas of engineering and applied science, including:

- Fourier analysis;
- digital signal processing;
- quantum mechanics;
- machine learning;
- image processing.

For example, every signal

$\boldsymbol{f(t)}$can be represented in terms of an orthogonal basis as

$$\boldsymbol{f(t)} = \sum_{\boldsymbol{n=1}}^{\infty} \boldsymbol{c_n\, \varphi_n(t)}.$$

This representation forms the mathematical foundation of Fourier series and many modern signal-processing techniques.

3.Pratical importance and application of the problem

The urgency of the problem: Functional analysis is one of the fundamental components of modern engineering education. This field of mathematics provides the theoretical foundation for key areas such as differential equations, optimization, signal processing, control theory, artificial intelligence, machine learning, and computational mathematics. Therefore, the high-level teaching of functional analysis in engineering programs is considered an essential requirement for the preparation of professional engineers.

The results of the study indicate that traditional lecture-based teaching of functional analysis is not sufficient for students to achieve a deep understanding of the subject. Constructivist learning, problem-based learning, project-based learning, research-oriented activities, and digital technologies significantly enhance the effectiveness of teaching functional analysis.

Scientific novelty of the problem: The integration of engineering problems into the educational process enables the demonstration of the practical relevance of functional analysis concepts and increases student motivation. The proposed teaching model combines theoretical training, computer-based modeling, project work, and research components, thereby meeting the requirements of modern engineering education.

Thus, the teaching of functional analysis in engineering programs plays a crucial role in developing future engineers’ analytical thinking, mathematical modeling abilities, and innovative problem-solving skills, and continued methodological research in this direction is essential.

The practical importance of the problem: The main objective of the proposed functional analysis teaching model for engineering programs is to integrate the theoretical foundations of functional analysis with engineering problems in order to develop students':

- theoretical knowledge;
- practical skills;
- research competencies.

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K.H.Aliyeva, L.V.Nasirova, U.R.Babayeva

Teaching functional analysis through engineering problems
Summary

This article investigates the methodological foundations of teaching functional analysis in engineering education and analyzes the impact of modern pedagogical approaches on students' learning outcomes. Constructivist learning, problem-based learning, project-based learning, inquiry-based learning, and digital technologies are examined as effective approaches for teaching functional analysis. Furthermore, the integration of functional analysis topics with engineering applications is discussed, and several instructional models and assessment strategies are proposed.

K.H.Əliyeva, L.V.Nəsirova, U.R.Babayeva

Funksional analizin mühəndislik problemləri vasitəsilə tədrisi
Xülasə

Məqalədə mühəndislik ixtisaslarında funksional analizin tədrisinin metodoloji əsasları araşdırılmış, müasir pedaqoji yanaşmaların bu fənnin mənimsənilməsinə təsiri təhlil edilmişdir. Konstruktiv təlim, problem əsaslı öyrənmə, layihə əsaslı öyrənmə, tədqiqat əsaslı təlim və rəqəmsal texnologiyaların funksional analizin tədrisində tətbiq imkanları nəzərdən keçirilmişdir. Eyni zamanda funksional analiz mövzularının mühəndislik problemləri ilə inteqrasiyası yolları göstərilmiş, müxtəlif tədris modelləri və qiymətləndirmə strategiyaları təklif edilmişdir.

К.Х.Алийева, Л.М.Насирова, У.Р.Бабайева

Преподавание функционального анализа через инженерные задачи

Резюме

В статье исследованы методологические основы преподавания функционального анализа в инженерных специальностях, а также проанализировано влияние современных педагогических подходов на усвоение данной дисциплины. Рассмотрены возможности применения конструктивистского обучения, проблемно-ориентированного обучения, проектно-ориентированного обучения, исследовательского обучения и цифровых технологий в процессе преподавания функционального анализа. Кроме того, показаны пути интеграции тем функционального анализа с инженерными задачами, предложены различные модели обучения и стратегии оценивания результатов обучения.

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